# Rhythmic Phase Symmetry, Inversion, and Reference Placement

A Framework for Relational Timing in Continuous Rhythmic Motion

F. Higuchi

September 1, 2026 · Working Paper — Version 1.2

## Status

This paper presents an exploratory framework for describing rhythmic timing as a relation between discrete musical events and continuous periodic phase. The formal temporal geometry developed below follows from explicit definitions. Claims concerning perception, sensorimotor organization, bodily movement, groove, pocket, stability, cultural style, or genre are not established by that geometry and are treated separately as empirical hypotheses.

The purpose of the framework is therefore not to define musical experience mathematically, but to identify a set of formal temporal relations that may be available for perceptual and motor recruitment.

No claim of priority over existing theories of rhythm, meter, entrainment, or embodied timing is intended.

## Research Provenance and Use of AI

The observations and theoretical framework presented in this paper originated from the author's independent development of and experimentation with SWING ARC. The central mathematical relations, interpretations, hypotheses, and conclusions were developed by the author through that process.

The sequence of development was:

1. development and practical experimentation with SWING ARC;
2. observation of recurring phase relationships while Beat and Subdivision timings were held fixed and continuous motion was displaced;
3. formulation and revision of the theoretical framework;
4. use of large language models as editorial and research-assistance tools.

Large language models were used to assist with **drafting and rewriting prose**, English translation, structural editing, mathematical formatting, literature discovery, bibliographic organization, and comparison with related research. They were not used as an autonomous source of empirical observations, and they are not treated as an authority for the theoretical claims made in this paper.

All theoretical claims, interpretations, selections, revisions, and final conclusions were determined and approved by the author.

The discussion of related literature should be understood as an **AI-assisted, non-systematic map of neighboring research**, not as a systematic or exhaustive literature review. Bibliographic details for works cited in the paper are intended to be checked against identifiable scholarly sources, but the selection, framing, and interpretation of the literature remain subject to independent scholarly review.

Responsibility for the arguments, interpretations, possible errors, and conclusions in this paper remains entirely with the author.

## How This Framework Emerged in SWING ARC

SWING ARC was initially developed as a visual metronome and practice tool. Its audible Beat and Subdivision timings could remain fixed while the phase of the continuous visual motion was displaced independently using an OFFSET control.

Repeatedly changing only the motion phase, without changing the sounding rhythm, produced a simple but important observation: some apparently different reference placements were related to one another by regular geometric operations.

This suggested that event timing and continuous reference placement could be treated as separate variables.

A second observation subsequently simplified the framework further. Several positions that initially appeared to require different explanations could be expressed through a single cyclic phase coordinate. In particular, the two midpoint relations surrounding a selected Subdivision satisfy

$$A_2 = A_1 + \frac{1}{2}.$$

Thus quarter-cycle placement, half-cycle displacement, and their combinations need not automatically be interpreted as independent mechanisms. They may instead represent structurally different positions produced by the same general operation: continuous phase displacement.

SWING ARC is therefore the discovery and experimental context of the framework, not a prerequisite for the theory itself. OFFSET is not the theory. It is simply a control that made relative phase easier to observe.

## Abstract

Rhythm is commonly described through the timing of discrete events such as Beats, Subdivisions, accents, and note onsets. This paper considers an additional structural variable: the placement of a continuous periodic phase relative to those fixed events.

Let two successive Beats be normalized to $0$ and $1$, and let a selected Subdivision occur at $S$, where $0 < S < 1$. The midpoint between the first Beat and the Subdivision is

$$A_1 = \frac{S}{2},$$

while the midpoint between the Subdivision and the following Beat is

$$A_2 = \frac{S+1}{2}.$$

These satisfy

$$A_2 = A_1 + \frac{1}{2},$$

showing that the two midpoint relations form a half-cycle pair within a single cyclic phase system.

The same construction applies to straight and swung subdivisions, ternary grids, and larger temporal levels. The resulting coordinates are not proposed as universal musical constants; they are consequences of selected temporal relations.

The paper first develops this formal geometry independently of musical interpretation. It then considers possible implications for perceptual locking, bodily organization, pocket, hierarchical 1&3 / 2&4 placement, stylistic preferences, and culturally specific rhythmic practices. These later claims are explicitly treated as hypotheses rather than consequences of the geometry.

## Architecture: From Formal Geometry to Human Behavior

The framework distinguishes four levels.

**Level 1: Formal Relational Geometry** concerns temporal relations defined in normalized cyclic time. These relations exist independently of sensory modality or musical interpretation.

**Level 2: Sensory Instantiation** concerns how those relations are presented through auditory, visual, tactile, proprioceptive, or other signals.

**Level 3: Cognitive / Sensorimotor Recruitment** concerns whether a formally available structure is actually used for attention, prediction, synchronization, or bodily organization.

**Level 4: Motor Expression** concerns how recruited temporal organization appears as tapping, force, sway, gesture, muscle activity, or other observable movement.

No one-to-one equivalence between these levels is assumed.

A geometrically available relation need not be perceived. A perceived relation need not be used as an internal timing reference. An internal reference need not appear transparently in observable movement.

This distinction is central to the framework.

# Part I — Formal Core

## 1. Fixed Rhythm, Movable Continuous Phase

Begin with a simple constraint: the rhythmic events remain fixed.

The Beat does not move. The Subdivision does not move. The sounding onset pattern does not move.

Only the phase of a continuous periodic reference is displaced around them.

Let the phase coordinate be $\phi$. The most general operation considered here is therefore

$$\phi \to \phi + \Delta$$

with phase understood cyclically modulo 1.

A displacement of $1/4$, $1/2$, or $3/4$ is not a different mathematical operation. Each is a movement along the same cyclic coordinate.

What differs is the relation encountered at each position.

This allows event timing and continuous phase placement to be treated as distinct variables.

## 2. Direct Alignment and Its Half-Cycle Counterpart

The simplest reference placement is direct alignment between a Beat and a selected Phase Landmark of the continuous cycle.

Let

$$D_1 = 0.$$

Its half-cycle counterpart is

$$D_2 = D_1 + \frac{1}{2} = \frac{1}{2}.$$

Thus direct alignment naturally produces the pair

$$0 \leftrightarrow \frac{1}{2}.$$

The second position does not require a new mechanism. It is the half-cycle counterpart of the first within the same cyclic phase coordinate.

Whether this half-cycle relation is experienced as inversion, reversal, opposing bodily direction, or something else is not determined by the geometry.

## 3. The Core Midpoint Construction

Normalize two successive Beats to

$$B_0 = 0, \qquad B_1 = 1.$$

Let a selected Subdivision occur at

$$0 < S < 1.$$

The midpoint between the first Beat and the Subdivision is

$$A_1 = \frac{S}{2}.$$

The midpoint between that Subdivision and the following Beat is

$$A_2 = \frac{S+1}{2}.$$

But

$$A_2 = \frac{S}{2} + \frac{1}{2},$$

and therefore

$$\boxed{A_2 = A_1 + \frac{1}{2}}.$$

This identity is central to the framework.

The two midpoint relations should not automatically be interpreted as independent special positions. They belong to the same relational family and are separated by exactly one half-cycle.

## 4. Relational Axis and Phase Landmark

A **Relational Axis** is a temporal relation defined from rhythmic events.

It is not inherently a physical location in space and it is not inherently a turning point or extremum.

A **Phase Landmark** is a reproducible feature of a continuous periodic process that can be aligned with such a temporal relation.

A Phase Landmark might be a turning point, positional extremum, zero crossing, maximum-velocity point, directional transition, or another reproducible phase feature.

More generally, continuous motion may be written as

$$x = g(\theta),$$

with phase progression

$$\theta = h(t).$$

Neither function is required to describe simple harmonic motion.

The formal geometry therefore does not depend on a sine wave, a pendulum, or any particular bodily gesture.

The useful principle is:

> **Choose the relation first; the coordinate follows.**

## 5. Coordinates Are Consequences, Not the Theory

Values such as 25%, 50%, 66.7%, 75%, or 83.3% are not fundamental constants of the framework.

They are coordinates produced by particular choices of temporal level, Subdivision, relation, and phase placement.

For example, in the negative OFFSET convention used in SWING ARC, a normalized phase position $A$ may be represented approximately as

$$\text{OFFSET} = -100A\%.$$

The negative sign is a display convention.

The theoretical object is the cyclic relation, not the interface coordinate.

# Part II — Straight, Swing, and Ternary Structure

## 6. Straight Eighth Notes

For straight eighth notes,

$$S = \frac{1}{2}.$$

Therefore

$$A_1 = \frac{1}{4}$$

and

$$A_2 = \frac{3}{4}.$$

Together with direct alignment and its half-cycle counterpart, this produces

$$0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}.$$

These positions form two half-cycle pairs:

$$0 \leftrightarrow \frac{1}{2}$$

and

$$\frac{1}{4} \leftrightarrow \frac{3}{4}.$$

In the negative OFFSET convention they appear approximately as

$$0\%, -25\%, -50\%, -75\%.$$

This provides a simple interpretation of the relationship between −25% and −75%.

They are not two unrelated special points.

Rather,

$$\frac{3}{4} = \frac{1}{4} + \frac{1}{2}.$$

The Beat–Subdivision midpoint has simply been moved to its half-cycle counterpart.

## 7. Continuous Swing

Swing moves the Subdivision, not the rule.

Suppose the swung Subdivision occurs at $S$. The same equations remain valid:

$$A_1 = \frac{S}{2}$$

and

$$A_2 = \frac{S}{2} + \frac{1}{2}.$$

If

$$S = .60,$$

then

$$A_1 = .30, \qquad A_2 = .80.$$

If the Swing ratio reaches the exact 2:1 triplet position,

$$S = \frac{2}{3},$$

then

$$A_1 = \frac{1}{3}$$

and

$$A_2 = \frac{5}{6}.$$

Straight and Swing therefore belong to one continuous geometric family.

There is no need to introduce a separate geometric rule when the Swing ratio changes.

## 8. Tempo: Same Geometry, Different Absolute Time

The geometry above is normalized, but human sensory and motor systems operate in physical time.

If tempo is $B$ Beats per minute, the duration of one Beat is

$$T = \frac{60}{B}.$$

A normalized position $p$ therefore corresponds to

$$t = pT.$$

The half-cycle separation in physical time is

$$\Delta t_{1/2} = \frac{T}{2} = \frac{30}{B}.$$

Thus two configurations may be geometrically identical while producing very different absolute temporal distances at different tempi.

This distinction becomes important when considering perception or motor behavior. At sufficiently high tempi, two phase relations that are easily distinguishable at slower tempi may become difficult to separate behaviorally.

The framework therefore assumes no universal BPM-independent perceptual threshold.

## 9. The Full Triplet Grid

A complete eighth-note triplet divides the Beat into

$$0, \frac{1}{3}, \frac{2}{3}, 1.$$

Unlike a binary eighth-note division, this grid contains two internal positions.

Applying the midpoint construction to $S = 1/3$ gives

$$\frac{1}{6}$$

and its half-cycle counterpart

$$\frac{2}{3}.$$

Applying the same construction to $S = 2/3$ gives

$$\frac{1}{3}$$

and

$$\frac{5}{6}.$$

The complete midpoint structure is therefore

$$\frac{1}{6} \leftrightarrow \frac{2}{3}$$

and

$$\frac{1}{3} \leftrightarrow \frac{5}{6}.$$

In SWING ARC notation, these correspond approximately to

$$-16.7\% \leftrightarrow -66.7\%$$

and

$$-33.3\% \leftrightarrow -83.3\%.$$

The triplet grid is structurally richer not because those values are intrinsically special, but because the ternary grid supplies an additional internal Subdivision to which the same midpoint construction can be applied.

## 10. Ordinary Triplet Swing and the Full Ternary Grid

An important distinction must be preserved.

Ordinary eighth notes swung at the exact 2:1 triplet ratio produce audible events at

$$0 \to \frac{2}{3} \to 1.$$

The $1/3$ position is not sounded.

Nevertheless, the same sounding rhythm is exactly commensurate with the complete ternary grid

$$0, \frac{1}{3}, \frac{2}{3}, 1.$$

This means that the unsounded $1/3$ position becomes a mathematically available internal reference.

That availability does not establish that performers or listeners actually recruit it.

It only makes such recruitment a testable possibility.

# Part III — Orientation, Hierarchy, and Nested Cycles

## 11. Relational Orientation and Inversion

The same Beat and Subdivision may support different functional orientations:

$$\text{Beat} \to \text{Subdivision}$$

versus

$$\text{Subdivision} \to \text{Beat}.$$

Nothing in the physical event order needs to reverse.

The score remains unchanged. The sounding events remain unchanged.

The distinction concerns how those events may be internally or bodily organized.

Likewise, for any position $P$,

$$P^* = P + \frac{1}{2} \pmod 1$$

defines its formal half-cycle counterpart.

A performer might experience this change as an inversion of movement direction or temporal orientation, but that interpretation is not built into the formal geometry.

Half-cycle displacement is mathematically certain.

Perceptual inversion is an empirical question.

## 12. Large Motion and Small Motion

The same temporal structure may be represented through nested continuous cycles.

A larger motion may correspond to a slower pulse, metrical organization, or bar-level cycle, while a smaller motion may represent Subdivision-level organization.

The relative phase between the two can change while the sounding event pattern remains fixed.

Conceptually:

Large Motion + Small Motion

may become

Large Motion + Phase-shifted Small Motion.

A compound or double-pendulum image is useful here because it makes nested periodic organization intuitive.

It should not, however, be confused with the theory itself.

The framework does not require rhythm to be physically generated by two pendulums. The pendulum is one possible representation of a more general relation between nested periodic processes.

## 13. Scale-Free Construction

The normalized interval $0 \to 1$ need not represent one quarter-note Beat.

It may represent a Subdivision cycle, one Beat, two Beats, a bar, or another selected periodic level.

The same relational construction can therefore be applied recursively.

> **Change the temporal level; keep the relational construction.**

This makes temporal hierarchy an essential part of the framework.

## 14. Local and Bar-Level Half-Cycle Symmetry

Half-cycle symmetry can occur at multiple temporal scales, but those scales must not be conflated.

At a local Beat level,

$$P \leftrightarrow P + \frac{1}{2}$$

describes two positions inside that selected Beat cycle.

In a four-Beat bar, however, a slower two-Beat organization produces two alternating Beat classes:

$$1\&3$$

and

$$2\&4.$$

A local Beat–Subdivision relation can remain unchanged while its placement inside this slower cycle changes:

$$1\&3 \leftrightarrow 2\&4.$$

This is not a new midpoint formula.

It is a change in **hierarchical placement**.

The distinction between local phase relation and higher-level placement becomes especially important when discussing backbeat-oriented music.

## 15. Meter, Compound Meter, and Irregular Grouping

A meter label does not uniquely determine the active temporal reference.

For example, 12/8 may be organized through its four large Beats, the ternary Subdivisions inside those Beats, pairs of Beats, the full bar, or combinations of several levels.

Similarly, 9/8 may be organized as 3+3+3, 2+2+2+3, or through other culturally and musically meaningful groupings.

Irregular meter adds another possibility: the relevant local reference structure may change across different regions of the bar.

The proper procedure is therefore not to infer phase organization directly from a meter name.

The temporal level and grouping must first be identified.

## 16. Relative Amplitude, Coupling, and Stability

Phase is unlikely to be the only variable relevant to musical feel.

Nested periodic structures may also differ in relative amplitude, coupling strength, coupling direction, or stability.

A useful exploratory formulation is therefore

$$\text{Feel} = f(\text{Phase Organization}, \text{Hierarchy}, \text{Relative Amplitude}, \text{Coupling}, \text{Stability}).$$

This is not proposed as an established quantitative law.

It simply identifies a broader set of variables that may matter once the formal phase structure is mapped onto human movement and perception.

# Part IV — Scope and Scientific Guardrails

## 17. What the Geometry Establishes

The formal framework establishes that fixed rhythmic events can be embedded within a movable continuous phase; that a selected Subdivision $S$ generates a midpoint

$$A_1 = \frac{S}{2};$$

that its adjacent counterpart satisfies

$$A_2 = A_1 + \frac{1}{2};$$

that direct Beat alignment likewise forms a half-cycle pair; that Straight and continuously variable Swing follow from the same construction; that ternary grids generate multiple midpoint families; and that the same geometry can operate at multiple temporal scales.

It also establishes that local phase relations and larger-cycle placement are conceptually distinct.

Nothing beyond these formal relations follows automatically.

## 18. What the Geometry Does Not Establish

The geometry does not show that one reference point is easier to perform.

It does not show that one position is inherently more groovy.

It does not prove that a coordinate uniquely defines a musical genre.

It does not prove that half-cycle displacement must be perceived as inversion.

It does not prove that any particular configuration produces pocket.

And it does not prove that every mathematically available reference relation is cognitively recruited by human performers or listeners.

Those are empirical questions.

## 19. Latent Structures: Mathematical Availability Is Not Cognitive Recruitment

Particular care is required when unsounded positions are introduced.

If every mathematically imaginable hidden grid were allowed after an observation had already been made, almost any result could be explained retrospectively.

Candidate latent structures should therefore be constrained in advance.

Low-order, commensurate structures should normally be tested before more complicated alternatives. Musical context, meter, accompaniment, repeated accents, stylistic practice, or explicit training may provide independent support for one candidate over another. Competing interpretations should be specified prospectively whenever possible.

The essential principle is:

> **Mathematical availability does not imply cognitive recruitment.**

## 20. Sensory and Experimental Considerations

Formal event time should not automatically be equated with raw acoustic onset.

Real sounds have attack, rise time, peak, sustain, decay, and offset. Perceived event time may therefore be better approximated by a perceptual center than by the physical beginning of the waveform.

Likewise, the geometry is not inherently visual merely because SWING ARC presents it visually. Similar structures could in principle be instantiated through audition, proprioception, touch, or self-generated motion.

Finally, observable movement is not a transparent readout of internal timing. A finger tap reflects motor preparation, muscle activation, force production, biomechanics, and surface interaction. EMG onset likewise does not directly reveal intention.

Strong tests should therefore distinguish sensory event time, internal temporal organization, and visible motor expression.

# Part V — AI-Assisted Map of Related Literature

*This section is a non-systematic, AI-assisted map of neighboring scholarship. It is intended to locate the present framework in relation to existing research, not to claim exhaustive coverage of the literature. Works cited here are listed formally in the References section.*

The present framework is not intended as a replacement for existing theories of rhythm perception, entrainment, meter, microtiming, or embodied musical cognition. Its proposed contribution is narrower: it isolates **continuous reference placement relative to fixed rhythmic events** as a formal variable and derives a small family of relational phase structures from that variable.

Several established research traditions are therefore directly relevant, but they address different parts of the problem.

## Dynamic Attending, Oscillatory Entrainment, and Neural Resonance

Dynamic Attending Theory describes temporal attention in terms of internal oscillatory processes that can entrain to external event structure and concentrate attentional energy at expected moments in time (Large & Jones, 1999). Related neural-resonance approaches propose that nonlinear neural oscillations can support the emergence of pulse and meter from complex rhythmic input. Experimental work has also reported neural responses corresponding to Beat- and meter-related periodicities, including metrical structure that is not reducible to a single acoustic onset sequence (Nozaradan et al., 2012).

The present framework is compatible with the general idea that rhythmic organization involves phase, oscillation, and entrainment, but it does not identify its continuous phase coordinate with a neural oscillator. **Formal phase geometry and physiological implementation are deliberately separated.**

The specific question introduced here is different: if external rhythmic events remain fixed, what formally defined relations become available when a continuous reference is displaced around them? Dynamic attending and neural resonance concern possible mechanisms by which temporal regularity is tracked or generated; the present framework attempts to specify additional **relational positions within that temporal organization** that such systems could, in principle, recruit.

## Meter, Temporal Hierarchy, and Multiple Periodic Levels

Research on beat and meter has long treated musical time as hierarchically organized rather than as a single undifferentiated pulse. Neural entrainment studies have reported responses at Beat and metrical levels (Nozaradan et al., 2012), while work on rhythmic attending has emphasized the ability to shift attention among faster and slower temporal levels.

This literature is closely related to the present distinction between **local Beat–Subdivision geometry** and **larger-cycle placement**. The current proposal adds a more explicit separation between two questions:

1. What is the local phase relation inside a selected temporal interval?
2. Where is that local relation placed within a slower periodic hierarchy?

This distinction is especially important for the later comparison between 1&3 and 2&4 organization in 4/4.

## Embodied Meter and Music-Induced Movement

A substantial body of work supports a close relation between rhythmic perception and bodily movement. Phillips-Silver and Trainor (2007) showed that bodily movement can influence how an ambiguous auditory rhythm is subsequently perceived metrically. Burger et al. (2014), using full-body movement, distinguished **period locking** from **phase locking** and demonstrated that spontaneous movement can organize around different metrical levels.

These findings support the need to distinguish discrete acoustic events from continuous bodily organization. They do not establish the midpoint relations proposed in the present paper, but they make the broader mapping problem empirically plausible: a temporal structure may be instantiated and recruited through continuous movement rather than only through discrete tapping.

The present terms **Relational Axis** and **Phase Landmark** are intended to make this connection more precise without assuming that a particular bodily trajectory—such as a pendulum endpoint—is universal.

## Swing Ratio and Tempo Dependence

Jazz swing research provides a particularly clear example of why rhythmic relations should not be reduced to fixed numerical constants. Friberg and Sundström (2002) found substantial variation in performed swing ratio with tempo, with long-short ratios tending to decrease as tempo increased.

This is consistent with two claims of the present framework. First, **the relation should be primary and the coordinate secondary**: a musical practice need not be defined by one fixed Swing percentage. Second, normalized phase geometry and absolute physical time must be distinguished. A structurally comparable phase relation may occupy very different millisecond durations at different tempi.

The present framework therefore does not treat Straight, Swing, and Triplet as isolated timing categories. It treats them as points or regions within a continuous family generated by the changing location of the Subdivision $S$.

## Microtiming, Participatory Discrepancies, and Groove

The relation between expressive timing deviation and groove has a long and contested history. Keil's concept of **participatory discrepancies** emphasized small processual and textural discrepancies as central to participatory musical experience (Keil, 1987). Later empirical work has examined whether performed microtiming deviations enhance groove. Senn et al. (2016), for example, found that both quantized and originally performed timing could receive high groove ratings, while exaggerated deviations reduced groove ratings.

This literature is especially important because the present framework introduces a distinction that conventional microtiming descriptions do not always make explicit:

**event displacement and reference displacement are not the same operation.**

Most microtiming experiments manipulate when notes are played relative to a metronomic grid. The SWING ARC observation that motivated this paper did the reverse: the audible events could remain fixed while only the continuous reference moved.

The framework therefore does not deny microtiming. Instead, it asks whether some experiences conventionally described as ahead, behind, laid-back, pushed, or pulled may sometimes involve a change in **reference organization**, rather than—or in addition to—a literal displacement of event onsets.

This distinction also provides a possible reason why a fully quantized performance need not be devoid of groove: the relevant organization may not be recoverable from onset deviation alone.

## Syncopation, Prediction, and the Desire to Move

Syncopation research addresses another neighboring but distinct dimension. Witek et al. (2014) found an inverted-U relationship between syncopation and both pleasure and the desire to move, with intermediate degrees of syncopation producing the strongest responses.

Such findings demonstrate that groove is sensitive to structural relations among events, not merely to local onset accuracy. However, syncopation changes or reweights the event structure itself. The present framework asks a complementary question: **can perceived or embodied organization change even when that event structure is held constant?**

A complete theory of groove may therefore need to consider both event structure—such as syncopation—and the continuous phase organization within which that structure is recruited.

## Perceptual Centers and the Definition of Event Time

The formal symbols $0$, $S$, and $1$ refer to selected temporal landmarks, but acoustic onset is not necessarily identical to perceived event time. Research on perceptual centers, or P-centers, shows that the perceived temporal location of a musical sound can differ systematically from its physical onset and depends on acoustic properties such as attack and rise time (London et al., 2019).

This literature directly motivates the framework's separation between **Formal Relational Geometry** and **Sensory Instantiation**.

If human synchronization is organized around perceptual event time rather than raw waveform onset, then experimentally measured Relational Axes should be constructed from the perceptually operative landmarks. The geometry itself remains unchanged; the sensory coordinates inserted into it may shift.

## Backbeat, Metric Layers, and 1&3 / 2&4 Organization

Music-theoretical research on groove-based popular music provides an important precedent for treating 2 and 4 as more than accidental accents on metrically weak Beats. Biamonte (2014), drawing on earlier work, argues that the backbeat can function as a contextually consonant and normative component of meter in rock and related traditions, and discusses temporal organization as a hierarchy of subtactus, tactus, metric, and hypermetric levels.

This is close to, but not identical with, the present proposal. The current framework does not redefine 2 and 4 as universally strong Beats. Instead, it asks whether the **same local phase relation** can occupy alternative positions within a slower two-Beat organization:

$$1\&3 \leftrightarrow 2\&4.$$

The aim is to distinguish surface accent, local Beat–Subdivision organization, and higher-level phase placement rather than collapse them into a single metric hierarchy.

This distinction may provide a formal way to investigate why a backbeat can be simultaneously oppositional to one metric description and completely normative within a musical practice.

## Embodied and Culturally Situated Timing in African American Music

The strongest connection to the cultural discussion later in this paper is the work of Vijay Iyer. Iyer (2002) explicitly brought embodied and situated cognition to the analysis of expressive microtiming in African American music, arguing that bodily universals are shaped by culturally specific practices and that rhythmic timing must be understood through both embodiment and sociocultural context.

That position is fundamentally compatible with the caution adopted here. The present geometry is not intended to reduce African American rhythmic practices to a set of universal phase coordinates. Rather, it proposes a **formal space of possible relations** whose actual recruitment can be culturally learned, bodily instantiated, and stylistically distributed.

Keil's participatory-discrepancy account similarly emphasizes musical timing as an active, socially participatory process rather than as error around an abstract grid (Keil, 1987).

The present framework therefore adds one candidate variable to an already rich field: **hierarchical phase placement relative to fixed rhythmic relations**. Whether that variable is actually recruited in Blues, Jazz, Gospel, Funk, Soul, R&B, Hip-Hop, or any other practice must be established empirically and historically, not inferred from geometry alone.

## Position of the Present Framework

Taken together, existing research already establishes several points that are essential to this paper:

- rhythmic cognition can involve oscillatory and phase-sensitive processes;
- Beat and meter can be represented at multiple temporal levels;
- bodily movement can influence rhythmic perception and can phase-lock to musical structure;
- Swing ratio varies with tempo;
- microtiming affects groove but is neither a simple necessary nor sufficient explanation of it;
- syncopation affects movement and pleasure;
- perceived event time may differ from acoustic onset;
- backbeat organization can be metrically normative within relevant musical practices;
- embodied rhythm is culturally situated.

The present paper does **not** claim priority over these findings.

Its proposed contribution is the narrower formal claim that fixed rhythmic events can be organized relative to a movable continuous phase, and that a selected Beat–Subdivision relation generates a midpoint and a half-cycle counterpart:

$$A_1 = \frac{S}{2}, \qquad A_2 = A_1 + \frac{1}{2}.$$

From this, the paper separates **local relational geometry**, **Phase Landmark selection**, and **higher-level hierarchical placement**.

That separation creates a specific empirical question that is not answered by the surrounding literature:

> **Can musicians and listeners recruit stable relations between events—not only the events themselves—as reference structures within a continuous or embodied phase?**

The Discussion that follows treats groove, pocket, pull, backbeat, genre, and cultural practice as possible domains in which that question can be tested, not as phenomena already explained by the formal construction.

# Part VI — Discussion: From Geometry to Musical Experience

Everything from this point onward is interpretive and empirical.

The geometry defines possible relations.

The following sections ask whether and how musicians may use them.

## 21. Locking to Relations, Not Only to Events

A common assumption in rhythmic synchronization is that one locks to discrete events.

The present framework suggests a broader possibility.

The midpoint between a Beat and a Subdivision may contain no audible onset, but it is not temporally undefined. It is reconstructed every cycle by its relation to the surrounding events.

If the same relation recurs repeatedly, a continuous Phase Landmark could in principle become aligned with it.

Synchronization may therefore occur not only to events themselves, but also to reproducible relations between events.

This provides one possible explanation for a familiar musical paradox: a performer may appear displaced from the explicit Beat while nevertheless feeling highly stable.

The relevant reference may not be a shifted Beat at all.

It may be a different relational axis.

## 22. Multiple Possible Phase Landmarks

Even if a relational axis is stable, the motion feature mapped onto it need not be universal.

One performer might organize the Beat around a turning point. Another might use a central crossing. Another might organize movement around a maximum-velocity region or directional transition.

This raises an important possibility.

Differences between performers or musical practices may involve not only **where** a continuous cycle is placed, but also **which feature of that cycle is treated as the operative landmark**.

Two performers could therefore share the same sounding rhythm while embodying it through different internal phase organizations.

## 23. Multiple Internal References and the Sense of Pull

A performer may also maintain more than one temporal reference simultaneously.

Suppose a Beat-centered reference remains active while another bodily or internal cycle is displaced toward a different relational axis.

The discrepancy between those references could be experienced as tension, pull, drag, forward pressure, backward pressure, or floating.

This may explain why some musicians describe groove in strongly directional language.

But that experience need not be universal.

A performer who acquired the displaced organization as a primary temporal reference may not experience any displacement at all.

What sounds “behind” to an outside observer may simply feel like normal time to the performer.

Thus:

$$\text{phase structure} \neq \text{subjective experience of phase difference.}$$

The sensation of being pulled may depend on the simultaneous retention of another reference.

## 24. Pocket as an Emergent State

Pocket is therefore better treated cautiously.

It need not be the fundamental mechanism that produces rhythmic organization.

A more conservative possibility is that pocket is an experiential state that may emerge when one or more performers stabilize within compatible temporal organizations.

A musician who naturally inhabits a particular organization may not consciously experience a distinct transition called “the pocket.”

Another musician attempting to imitate that organization may move from instability to stability and experience that transition very strongly.

In that sense,

$$\text{phase organization} \to \text{stable coordination} \to \text{possible experience of pocket.}$$

Pocket clearly depends on more than phase alone. Dynamics, articulation, note length, timbre, ensemble interaction, and musical context remain essential.

## 25. 1 & 3 and 2 & 4 as Hierarchical Placement

The distinction between local geometry and larger-scale placement provides another useful possibility.

In 4/4, a local Beat–Subdivision relation can remain identical while being embedded differently in a slower alternating cycle.

One placement may align with the 1&3 class; its half-cycle alternative may align with the 2&4 class.

This suggests that the musical importance of 2 and 4 need not always be understood only as an accent placed on those Beats.

A backbeat accent may sometimes be the audible surface expression of a deeper periodic organization.

That possibility is not established by the present geometry, but the framework makes it possible to formulate and test it directly.

## 26. Backbeat and "The One"

A 2&4-oriented higher-level organization does not imply that Beat 1 must become unimportant.

Beat 1 participates simultaneously in more than one temporal relation.

Within an alternating 1&3 / 2&4 organization, Beat 1 belongs to the 1&3 class.

But within the bar, Beat 1 also serves as a unique origin or reset point.

These functions can coexist.

It is therefore possible, in principle, for a musical practice to exhibit a strong 2&4-oriented periodic organization while simultaneously assigning exceptional structural importance to "The One."

This is not a contradiction.

It is a consequence of hierarchical timing: the same event can have different functions at different temporal levels.

## 27. Style and Genre as Preference Distributions

The formal geometry itself does not assign genres to coordinates.

There is no basis here for statements such as "Jazz is −X%" or "Funk is −Y%."

A more plausible hypothesis is that musical cultures and styles may differ statistically in which phase relations, hierarchical placements, Phase Landmarks, amplitudes, and coupling patterns they tend to recruit.

A genre might therefore be associated not with one correct coordinate but with a **distribution of preferred organizations**.

That distribution could vary with performer, tempo, historical period, instrumentation, musical context, or local tradition.

This formulation also leaves room for disagreement about what sounds stylistically convincing.

Two performances may use the same score and approximate onset pattern while differing in deeper temporal organization.

## 28. African American Musical Traditions

This question may be particularly relevant to musical traditions emerging from African American cultural practice.

Blues, Jazz, Gospel, Soul, Funk, R&B, and related traditions have repeatedly been described through concepts such as swing, backbeat, syncopation, laid-back timing, groove, and pocket.

These descriptions are musically useful, but they refer to phenomena at several different temporal levels.

The distinction proposed here between **local Beat–Subdivision relation** and **larger hierarchical placement** may offer another way to examine the prominent role of 2 and 4 across many such practices.

In particular, it raises the possibility that some manifestations of backbeat should be investigated not only as surface accent patterns, but also as expressions of a deeper phase organization at a slower temporal level.

The half-cycle relationship between structurally paired placements provides a formal reason why 1&3 and 2&4 organizations can be compared without treating them as unrelated timing systems.

This paper does not claim that African American rhythmic practice can be reduced to such geometry, nor that any coordinate defines those traditions.

Cultural transmission, articulation, timbre, interaction, movement, phrasing, history, and performance practice remain indispensable.

The narrower proposal is that hierarchical phase placement may be one additional variable worth examining.

### 29. FEEL as an Experimental Application

SWING ARC includes FEEL presets that combine tempo, Swing, phase placement, and accent organization.

These presets should not be interpreted as genre definitions.

They are practical hypotheses.

A FEEL configuration can propose one possible organization, allow a performer to experience it, and then be modified, rejected, or refined.

This creates a useful distinction between the paper and the application.

The paper defines relations and hypotheses.

The application provides a space in which concrete musical interpretations of those hypotheses can be explored.

### 30. Dance and Embodied Organization

Nested phase organization may also be relevant to dance and larger bodily movement.

The body does not necessarily represent every rhythmic level through the same gesture.

A larger center-of-mass movement may reflect meter or a slower periodic cycle, while feet, hands, shoulders, or other local gestures articulate Subdivision, accent, or groove.

This means that a body can simultaneously carry several temporal layers whose relative phases are not identical.

A large movement and a small movement may even appear to contradict one another locally while remaining stable as a combined organization.

The CAT motion experiments in SWING ARC are one practical attempt to visualize this possibility.

Again, they are experimental representations rather than evidence that the body must organize rhythm in precisely that way.

## Part VII — Testable Predictions

The framework generates several classes of prediction.

First, every local midpoint

$$A_1 = \frac{S}{2}$$

has a formal counterpart

$$A_2 = A_1 + \frac{1}{2}.$$

If one position produces a reproducible behavioral effect, its half-cycle counterpart becomes a natural comparison point.

Second, when Swing changes continuously, the predicted relational positions should move continuously with $S$. Straight and Swing should therefore not require categorically different geometric models.

Third, if exact triplet context causes performers to recruit the silent $1/3$ ternary position, strengthening or weakening ternary context while preserving the sounding onset sequence should selectively affect behavior associated with its derived midpoint family.

Fourth, changing only the continuous phase while preserving the score and sounding events may alter perceived naturalness, bodily organization, synchronization stability, or reported feel.

Fifth, local phase organization should be maintainable while larger hierarchical placement changes. In 4/4, a local structure might remain constant while its relationship to 1&3 versus 2&4 changes.

Sixth, changing relative movement amplitude or coupling while preserving timing may produce effects independent of phase placement.

Seventh, if the relevant organization depends on temporal hierarchy rather than meter labels alone, changing Subdivision or grouping while preserving the written meter should change which phase relations become salient.

Eighth, equivalent normalized phase relations may produce different behavioral results as tempo changes and absolute temporal intervals shrink.

Ninth, if perceived event time rather than acoustic onset determines phase placement, effects should track independently measured perceptual centers when onset and perceptual center are experimentally dissociated.

Tenth, if the structure is modality-general, related effects should survive changes between visual, auditory, proprioceptive, or other implementations.

The present Discussion also generates several additional predictions.

If performers can lock to relations rather than only to events, stable behavior should sometimes occur at reproducible Beat–Subdivision relational positions even when no onset occurs there.

If the subjective feeling of “pull” depends on maintaining competing internal references, performers who report strong pull should differ systematically from performers who experience the same phase organization as neutral.

If musical style recruits phase organization probabilistically, phase-preference distributions should differ across performer populations, training backgrounds, or musical practices without collapsing to a single genre coordinate.

Finally, if 1&3 / 2&4 organization reflects hierarchical placement rather than merely accent strength, changing the higher-level placement while preserving local timing should affect musical or bodily judgments even when the surface onset pattern is controlled.

## Conclusion

The framework begins with a simple operation:

**keep the rhythmic events fixed and move only the continuous phase around them.**

For a selected Subdivision $S$,

$$A_1 = \frac{S}{2}$$

and

$$A_2 = \frac{S+1}{2},$$

which gives

$$A_2 = A_1 + \frac{1}{2}.$$

Direct Beat alignment has the same half-cycle structure:

$$0 \leftrightarrow \frac{1}{2}.$$

For straight eighth notes, this produces

$$0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4},$$

which can be understood as two half-cycle families rather than four unrelated special positions.

The same construction extends continuously into Swing, becomes richer in ternary grids, and can be applied at multiple temporal levels.

The score need not shift.

The onset pattern need not shift.

The physical stroke pattern need not shift.

The continuous or embodied temporal reference may shift.

The formal question is therefore relatively simple: **what relational structures are available within cyclic phase?**

The scientific question is much larger: **which of those structures do human perception and movement actually recruit, under what musical conditions, and with what consequences for feel?**

That second question includes the possibility that performers synchronize not only to audible events but also to repeatable relations between them; that higher-level 1&3 / 2&4 placement interacts with local Beat–Subdivision organization; that pocket may emerge from stable coordination rather than constitute its cause; and that different musical traditions may preferentially recruit different regions or organizations of the same underlying phase space.

The geometry does not answer those questions.

It makes them more precisely askable.

## Citation and Reference Style

In-text citations and the reference list follow **APA 7th edition author–date style**. Bibliographic details and DOI identifiers have been checked against publisher, journal, PubMed, or other scholarly bibliographic records.

# References

Biamonte, N. (2014). Formal functions of metric dissonance in rock music. *Music Theory Online, 20*(2). https://doi.org/10.30535/mto.20.2.1

Burger, B., Thompson, M. R., Luck, G., Saarikallio, S. H., & Toiviainen, P. (2014). Hunting for the beat in the body: On period and phase locking in music-induced movement. *Frontiers in Human Neuroscience, 8*, Article 903. https://doi.org/10.3389/fnhum.2014.00903

Friberg, A., & Sundström, A. (2002). Swing ratios and ensemble timing in jazz performance: Evidence for a common rhythmic pattern. *Music Perception, 19*(3), 333–349. https://doi.org/10.1525/mp.2002.19.3.333

Iyer, V. (2002). Embodied mind, situated cognition, and expressive microtiming in African-American music. *Music Perception, 19*(3), 387–414. https://doi.org/10.1525/mp.2002.19.3.387

Keil, C. (1987). Participatory discrepancies and the power of music. *Cultural Anthropology, 2*(3), 275–283. https://doi.org/10.1525/can.1987.2.3.02a00010

Large, E. W., & Jones, M. R. (1999). The dynamics of attending: How people track time-varying events. *Psychological Review, 106*(1), 119–159. https://doi.org/10.1037/0033-295X.106.1.119

London, J., Nymoen, K., Langerød, M. T., Thompson, M. R., Code, D. L., & Danielsen, A. (2019). A comparison of methods for investigating the perceptual center of musical sounds. *Attention, Perception, & Psychophysics, 81*(6), 2088–2101. https://doi.org/10.3758/s13414-019-01747-y

Nozaradan, S., Peretz, I., & Mouraux, A. (2012). Selective neuronal entrainment to the beat and meter embedded in a musical rhythm. *The Journal of Neuroscience, 32*(49), 17572–17581. https://doi.org/10.1523/JNEUROSCI.3203-12.2012

Phillips-Silver, J., & Trainor, L. J. (2007). Hearing what the body feels: Auditory encoding of rhythmic movement. *Cognition, 105*(3), 533–546. https://doi.org/10.1016/j.cognition.2006.11.006

Senn, O., Kilchenmann, L., von Georgi, R., & Bullerjahn, C. (2016). The effect of expert performance microtiming on listeners' experience of groove in swing or funk music. *Frontiers in Psychology, 7*, Article 1487. https://doi.org/10.3389/fpsyg.2016.01487

Witek, M. A. G., Clarke, E. F., Wallentin, M., Kringelbach, M. L., & Vuust, P. (2014). Syncopation, body-movement and pleasure in groove music. *PLOS ONE, 9*(4), e94446. https://doi.org/10.1371/journal.pone.0094446